RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. MID SEMESTER EXAMINATION, SEPTEMBER 2012

SECOND YEAR

MATHEMATICS (Honours)

Date : 10/09/2012

Time : 2 pm – 4 pm

Paper : III

Full Marks : 50

[Use separate answer-books for each group]

Group–A

[*Answer **any five** questions*] 5x5

1. a) Let $V=\mathbb{R}^4$ and W be a subspace of V generated by the vectors (1,0,0,0), (1,1,0,0). Find a basis of the quotient space V/W. Verify that $\dim V/W = \dim V - \dim W$. 3

   b) If A be a non singular matrix, prove that the row vectors of A are linearly independent. 2

2. Find the row space and row rank of the matrix $\begin{pmatrix} 2 & 1 & 3 & 5 \\ 3 & 4 & 1 & 2 \\ 0 & 3 & 1 & 1 \\ 5 & 5 & 4 & 7 \end{pmatrix}$. 5

3. Investigate for what values of λ and μ the following equations:

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

   have (i) no solution, (ii) unique solution and (iii) an infinite number of solutions. 5

4. Let A, B be two matrices over the field $\mathbb{R}$ of real numbers such that AB is defined. Prove that rank of $(AB) \le \min\{\text{rank of } A, \text{ rank of } B\}$. 5

5. The matrix representation of a linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & 2 & -1 \end{pmatrix}$$

   relative to the standard ordered basis of $\mathbb{R}^3$. Find the explicit representation of T and matrix of T relative to the ordered basis $\{(0,1,2),(-1,0,1),(2,1,1)\}$. 5

6. Let V and W be vector spaces over the same field F. Prove that a linear transformation $T:V\to W$ is invertible if and only if T is an isomorphism. 5

7. Determine the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ that maps the basis vectors (0,1,1), (1,0,1), (1,1,0) of $\mathbb{R}^3$ to the vectors (2,1,1), (1,2,1), (1,1,2) respectively. Find Ker(T) and Im(T). Verify that $\dim(\mathrm{Ker}(T)) + \dim(\mathrm{Im}(T)) = 3$. 5

8. a) Let V and W be finite dimensional vector spaces of same dimension over the same field F and $T:V\to W$ be a linear transformation. Prove that T is one-to-one iff T is onto. 3

b) Give an example of a linear operation T on a vector space V over a field F such that $\mathrm{Ker}(T)=\mathrm{Im}(T)$. Justification needed. 2

Group–B

*Answer **any two** questions:* 2x5

9. The plane $ax+by+cz+d=0$ bisects an angle between a pair of planes one of which is $lx+my+nz+p=0$. Show that the equation of the other plane of the pair is $(lx+my+nz+p)(a^2+b^2+c^2)=2(al+bm+cn)(ax+by+cz+d)$. 5

10. A variable line intersects the lines

$$y=0, z=c; x=0, z=-c$$

and is parallel to the plane $lx+my+nz=p$. Prove that the surface generated by it is $lx(z-c)+my(z+c)+n(z^2-c^2)=0$. 5

11. Find the sphere with smallest radius which touches the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z-6}{1}$ and $\frac{x+3}{7}=\frac{y+3}{-6}=\frac{z+3}{1}$. 5

*Answer **any one** question:* 1x5

12. a) A weightless elastic string of natural length l and modulus λ, has two equal particles of mass m each at its ends and lies on a smooth horizontal table perpendicular to an edge with one particle just hanging over. Show that the other particle will pass over at the end of time t given by the equation $2l+\frac{mgl}{\lambda}\sin^2\sqrt{\frac{\lambda}{2ml}}t=\frac{1}{2}gt^2$. 8

b) A particle is projected with velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is k times the velocity. Show that its direction will again make an angle α below the horizontal after a time $\frac{1}{k}\log(1+\frac{2ku}{g}\sin\alpha)$. 7

13. a) If the earth's attraction varies inversely as the square of the distance from the centre and g be its magnitude at the surface; show that the time of falling of a particle from a height h above the surface to the surface is $\sqrt{\frac{a+h}{2g}}\left[\frac{a+h}{a}\cos^{-1}\sqrt{\frac{a}{a=h}}+\sqrt{\frac{h}{a}}\right]$, where a is the radius of the earth and the resistance of air is neglected. 8

b) A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length $l+c$ being at one side and $l-c$ at the other; if the end of the shorter portion be held and then let go, show that the chain will slip off the pulley in time $\sqrt{\frac{l}{g}}\log\frac{l+\sqrt{l^2-c^2}}{c}$, $(l>c)$. 7
